Ruin problem in a changing environment and application to the cost of climate change for an insurance company

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Abstract

In this paper we obtain asymptotics for ruin probability in a risk model where claim size distribution as well as claim frequency change over time. This is a way to take into account observed and/or projected changes, due to climate change, in some specific weather-related events like tropical storms for instance. Some examples will be presented in order to illustrate the theory and start a discussion on the possible cost of climate change for an insurance company who wants to remain financially solvent.

Keywords: Non homogeneous Poisson process, ruin probability, asymptotics, cost of climate change.

1 Introduction

Climate change is of great concern for insurers because of increasing in frequency and intensity of extreme weather events. Tropical cyclones, for example, represent a significant threat to coastal population and infrastructure as well as marine interests (shipping and offshore activities for example). More frequent and/or intensive events would affect insurance systems because of increasing losses also related to economic growth, a greater concentration of people and wealth in periled areas and rising insurance penetration.

Detection of long term past trends in measures of tropical cyclone activity is constrained by data availability, quantity and quality: for example, historical tropical cyclone records are known to be heterogeneous owing to changes in measurement practices over time. Nevertheless, the IPCC (Intergovernmental Panel on Climate Change) Fifth Assessment Report (AR5, WG1)([8]) highlighted that there is very strong evidence that frequency and intensity of the strongest tropical cyclones have increased in the North Atlantic basin since the 1970s. As to the future, a broad range of modeling studies, resumed in the IPCC Special Report on Extreme Events ([7]), project decreases or no change in the overall global tropical cyclone frequency

but also project a substantial increase in the frequency of the most intense storms in some ocean basins by the end of 21st century. This is clearly worrying as the strongest storms are generally responsible for the majority of damage.
IPCC Special Report on Extreme Events ([7]) also announces, because of climate change, a likely increase in mean maximum wind speed in tropical cyclones as well as in heavy rainfalls associated with tropical cyclones which is expected to intensify cyclone's impacts.

According to the IPCC Fifth Assessment Report ([8]), direct and insured losses from weather-related disasters have substantially increased in recent decades. With respect to the future, the projected increase in the frequency of most intensive tropical cyclones will result in higher direct economic losses and loss variability. This will challenge insurance systems to offer coverage for premiums which are still affordable while at the same time requiring more risk-based capital (AR5, WG2). It is clear that if insurance coverage is to be maintained, insurers would need more risk-based capital to indemnify catastrophic losses and remain financially solvent.

Apart from climate change related trends, tropical storms and hurricanes also exhibit very specific seasonal patterns. The peak season for these weather events in the Southern Hemisphere is January to March while in the Northern Hemisphere, most tropical storms and hurricanes develop in June to November. Figure 1 describes the yearly estimated probability of hurricane occurrence in the Florida gulf for the period 1851 to 2013. Clearly there is a pattern in such a kind of climatic event: tropical storms and hurricanes rarely form during the months December to April in the North Atlantic and North Pacific basins. Indeed, Atlantic basin tropical cyclone activity either before 1 June or after 30 November is nearly negligible. Both storms and hurricanes show a strong maximum in mid-September with most cyclones occurring between 1 August and 31 October, when hurricane season reaches its peak.
The same kind of seasonal pattern may also be observed for losses: in general, most expensive hurricanes will occur when the probability of occurrence of the hurricane is higher. For example, Katrina (125 Milliards dollars) occurred at the end of August, beginning of September (2005), Sandy (68M) and Wilma (29.3M) occurred at the end of October (2012 and 2005), Ike (37.5) occurred at the beginning of September (2008) and so on. Some exceptions are represented by Beryl (148 000 dollars, end of May 2012), Arthur (78 millions, end of May 2008), Olga (45 millions, mid December 2007) and Odette (8 millions, beginning of December 2003) but they were classified as tropical storms and not hurricanes.

In the light of the above comments, we will assume in our paper that not only frequencies but also observed losses are changing over time. For example, losses may exhibit seasonal variations corresponding to the meteorological characteristics of the considered weather events as well as an increasing trend due to climate change. Ignoring changing hazard conditions results in biased estimates of expected loss, loss variability and risk capital requirements and this may affect the solvency of an insurance company.

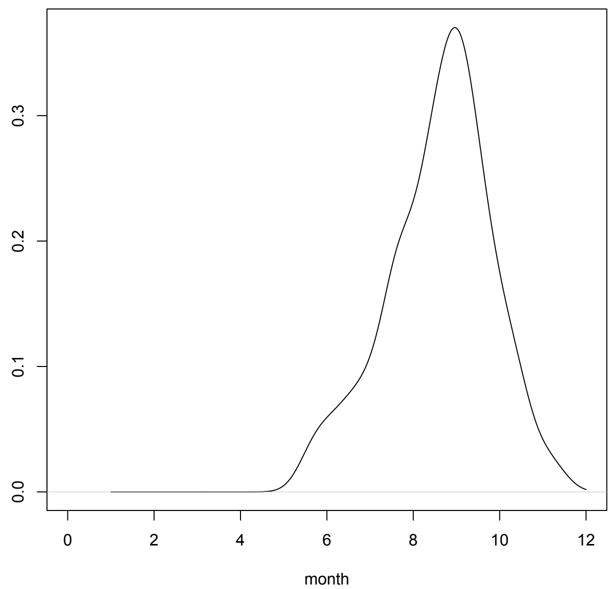

Figure 1: Estimated probability of hurricane occurrence in the gulf of Florida in one year conditionally to the occurrence of the hurricane (1851–2013).

Ruin problems in a periodic environment have been studied in Asmussen and Rolski (1994) [3] and in Rolski et al. (1999) [13]. A periodic risk process is studied in Chukova et al. (2000) [6] from a reliability theory point of view. A risk process with periodic claim intensity is considered by Morales (2004) [12] who provides a practical simulation methodology to evaluate ruin probabilities and Lu and Garrido (2005) [11] who study estimation method.

Lin (2014) [10] studies second order asymptotic results for the sum of not necessarily identically distributed heavy tailed random variables with applications to ruin theory. A time inhomogeneous risk model has also been studied in Kortschak et al. (2015) [9] where claim size distribution changes over time in an unfavorable direction but there is no change in claim frequency. In this paper we study the asymptotics of the ruin probability when both the frequency and the severity depend on time. Our aim is to start a discussion on the cost of climate change for an insurance company in a sense that will be specified later in the paper. Finding a model for climate change is not an easy task. Therefore we analyze a simplified insurance portfolio on a long term basis which in some sense resembles climate change. We will assume that the insurance company works on a yearly basis. This means that at the beginning of each year the company sets a premium that is adequate compared to the expected claims for the next period. Further the insurance company has to provide enough solvency capital in order to cover the signed risks and remain solvent.
In the long run we will expect that the paid claims are approximately the same

as the expected claims and this is reflected by an increased premium. Since it is natural to think that customers are willing to accept premiums that are related to their expected claim amounts, we will assume that the increase in claims is basically paid by the customers and hence is no real cost for the insurance company. On the other hand, because of the increased risk we will assume that the needed risk capital will increase over time. Now, this increase in capital is basically what the climate change will cost the insurance company.

The paper is organized as follows. In Section 2, we present a risk model that allows for frequency and severity to depend on time. Asymptotics for ruin probability under the proposed model are obtained in Section 3. In Section 4 we present some examples and focuse on the cost of adapting to the impacts of climate change for an insurance company. Conclusions are provided in Section 5.

2 The risk model

As mentioned above, we want to define a risk model which allows to take into account a changing environment due to climate change. Once the model defined and some results on ruin probability obtained, we will try to define a way to calculate the cost of climate change for the insurance company in a hypothetical situation.
Our mathematical framework is the following. Assume that $X_1, X_2, \dots$ are independent and identically distributed (iid) regularly varying random variables (rv's) of index $\alpha > 1$ and distribution function (df) F. The risk process will be defined as

$$S_t = \sum_{i=1}^{N_t} \mu(\mathcal{T}_i) X_i - ct$$

where N_t is a nonhomogeneous Poisson process with intensity $\lambda(t)$, μ is a bounded function on $[0, T]$, $\mathcal{T}_i$ is the time of the i-th claim and c is the premium intensity. Further denote with $Y_i = \mu(\mathcal{T}_i)X_i$ the i-th claim size, $T_i = \mathcal{T}_i - \mathcal{T}_{i-1}$ the inter arrival times, $\mu_m = \min_{0 \le t \le T} \mu(t)$, $\mu_M = \max_{0 \le t \le T} \mu(t)$ and $\Lambda(t) = \int_0^t \lambda(x) \mathrm{d}x$ the hazard function of the process. The function $\mu(\mathcal{T}_i)$, which is representing a multiplicative factor on the claim size, will be chosen in such a way to ensure a severity changing over time.
We are interested in the finite time ruin probability defined by

$$\psi(u, T) = \mathbb{P}\left(\sup_{0 \le t \le T} S_t > u \right)$$

where u is the insurer's initial capital. To evaluate $\psi(u, T)$ we will use an asymptotic approximation. Note that for fixed N_t, $\sum_{i=1}^{N_t} \mu(\mathcal{T}_i)X_i$ has the same distribution as the same sum where the $\mathcal{T}_i$'s are replaced by iid random variables (T_i) with density $\lambda(x)/\Lambda(t)$. Hence we can deduce a first order asymptotic approximation by standard methods like in [2, section X.4]

$$\psi(u, T) \sim \int_0^T \mathbb{P}(X_1 > u/\mu(x)) \lambda(x) \mathrm{d}x. \tag{1}$$

Since this asymptotic does not depend on the premium rate c, Section 3 will be devoted to the calculation of the error rate for this approximation. As we will see (Theorem 3.1), the error rate will depend on the premium rate. Indeed, this is important because we would like to integrate changes in premiums (due to changes in risk) in our calculations of the solvency capital for example (see Section 4). Since the error rate obtained in Theorem 3.1 is not explicitly given, bounds will be derived in Corollary 3.2 and the upper bound, given below, will be used as an approximation for the ruin probability in Section 4, for the applications:

$$\begin{aligned}
&\psi(u,T)\\
&\approx \mathbb{E}\left[\int_0^T \lambda(t)\overline{F}(u/\mu(t))\mathrm{d}t\right]\\
&\quad + f(u)\int_0^T \lambda(t)\left(\mu(t)\mathbb{E}\left[X_1\right]\int_0^t \mu(y)^\alpha\lambda(y)\mathrm{d}y + \mu(t)^\alpha\left(\mathbb{E}\left[X_1\right]\int_0^t \mu(x)\lambda(x)\mathrm{d}x - ct\right)\right)\mathrm{d}t.
\end{aligned}$$

3 Asymptotic Results

In this section we provide the approximation for the error term in the asymptotic approximation (1). Note that similar results can also be found in Borovkov and Borovkov (2002) [5] and Lin (2014) [10].

Theorem 3.1. *Let $X_1, X_2, \ldots$ be iid random variables with distribution function F and survival function $\overline{F}$. F is regularly varying of index $\alpha > 1$ and has regularly varying density f. Further let $0 < \mu_m \leq \mu(t) \leq \mu_M$ be a bounded function (on $[0,T]$). Then*

$$\begin{aligned}
&\psi(u,T) - \mathbb{E}\left[N_T\right]\int_0^T \mathbb{P}(X_1 > u/\mu(x))\frac{\lambda(x)}{\Lambda(T)}\mathrm{d}x\\
&\qquad \sim f(u)\Bigg(\int_0^T \lambda(t)\mu(t)^\alpha\left(\mathbb{E}\left[X_1\right]\int_0^t \lambda(x)\mu(x)\mathrm{d}x - ct\right)\mathrm{d}t\\
&\qquad\qquad + \int_0^T \lambda(t)\mu(t)\mathbb{E}\left[\sum_{k=1}^{N_t}\mu(\mathcal{T}_k)^\alpha\int_{\sup_{\mathcal{T}_k\leq s\leq t}\frac{c(t-s)-(S_{N_t,-k}-S_{N_s,-k})}{\mu(t)}}^\infty \overline{F}(y)\,\mathrm{d}y\right]\mathrm{d}t\Bigg).
\end{aligned}$$

Remark 3.1. *Lin (2014) [10] provides error rates for the probability that the maximum of the cumulative sums of independent but not identically distributed random variables exceeds a threshold u. This problem is quite close to the problem studied in this paper. Hence, we may write the error rate in Theorem 3.1 in a similar way to the rate provided in Lin (2014) [10]:*

$$\begin{aligned}
&f(u)\Bigg(\mathbb{E}\left[X_1\right]\mathbb{E}\left[\sum_{k=1}^{N_T}\mu(\mathcal{T}_k)^\alpha\sum_{i\neq k}^{N_T}\mu(\mathcal{T}_i)\right] - c\mathbb{E}\left[\mathcal{T}_{N_T}\sum_{k=1}^{N_T}\mu(\mathcal{T}_k)^\alpha\right]\\
&\qquad + \mathbb{E}\left[\sum_{k=1}^{N_T-1}\sum_{l=1}^{k}\mu(\mathcal{T}_l)^\alpha\left(0\vee\min_{k<m\leq N_T}\left(c(\mathcal{T}_m-\mathcal{T}_k) - \sum_{i=k+1}^{m}\mu(\mathcal{T}_i)X_i\right)\right)\right]\Bigg).
\end{aligned}$$

Proof. Without loss of generality (w.l.o.g.) we will assume that $\mu_M > 1$. Conditioning on the event that ruin occurs at time t we get

$$\psi(u,T) = \int_0^T \lambda(t)\mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\inf_{0\le s<t} u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0\right\}}\right]\mathrm{d}t.$$

To evaluate the above integral note that

$$\begin{aligned}
&\mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\inf_{0\le s<t} u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0\right\}}\right]\\
&= \mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\inf_{0\le s<t} u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0,\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i\le u/2\right\}}\right] \qquad (2)\\
&\quad + \mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\inf_{0\le s<t} u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0,\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>u/2\right\}}\right]. \qquad (3)
\end{aligned}$$

At first we analyze the term (2). Note that

$$1_{\left\{\inf_{0\le s<t} u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0,\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i\le u/2\right\}} = 1_{\left\{\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i\le u/2\right\}}.$$

Further a Taylor expansion leads to ($0 \le |\xi| \le |ct - \sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i|$)

$$\begin{aligned}
&\mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i\le u/2\right\}}\right]\\
&= \overline{F}(u/\mu(t))\mathbb{P}\left(\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i \le u/2\right) + \mathbb{E}\left[\frac{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i - ct}{\mu(t)} f\left(\frac{u-\xi}{\mu(t)}\right)1_{\left\{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i\le u/2\right\}}\right].
\end{aligned}$$

By Potter bounds ([4]) we get that there exists an $\epsilon > 0$ and a $C_1 > 1$ so that

$$\begin{aligned}
\overline{F}(u/\mu(t))\mathbb{P}\left(\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i > u/2\right) &\le \overline{F}(u/\mu(t))\mathbb{E}\left[N_T\mathbb{P}\left(X_1 > \frac{u}{2N_T\mu_M}\right)\right]\\
&\le C_1\overline{F}(u)^2\overline{\mu}^{2\alpha+\epsilon}\mathbb{E}\left[N_T^{1+\alpha+\epsilon}\right],
\end{aligned}$$

where $\overline{\mu} = \sup_{0\le s\le t}\mu(t)$. Since f is regularly varying we get for some $C_2 > 0$ the bound

$$\left|\frac{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i - ct}{\mu(t)} f\left(\frac{u-\xi}{\mu(t)}\right)1_{\left\{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i\le u/2\right\}}\right| \le C_2 f(u)(2\mu_M)^{1+\alpha+\epsilon}\left|\frac{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i - ct}{\mu(t)}\right|.$$

By dominated convergence we get that

$$
\begin{aligned}
&\mathbb{E}\left[\int_0^T \lambda(t)\mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i\le u+ct\right\}}\right]\mathrm{d}t\right]\\
&=\mathbb{E}\left[\int_0^T \lambda(t)\overline{F}(u/\mu(t))\mathrm{d}t\right]\\
&\quad+f(u)\int_0^T \lambda(t)\mu(t)^\alpha\mathbb{E}\left[\left(\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i-ct\right)\right]\mathrm{d}t+o\left(f(u)\right)+\mathcal{O}(\overline{F}(u)^2)\,.
\end{aligned}
$$

Next we have to provide the asymptotic expansion for (3). Note that for $i\neq j$

$$
\mathbb{P}\left(X_i>\frac{1}{4\mu_M N_T},X_j>\frac{1}{4\mu_M N_T}\right)\le C_3\mathbb{E}\left[N_T^{2\alpha+\epsilon}\right]\mu_M^{2\alpha+\epsilon}\overline{F}(u)^2.
$$

Denote with $S_{N_s,-k}=\sum_{i=1,i\neq k}^{N_s}\mu(\mathcal{T}_i)X_i$ and

$$
A_k=\left\{\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i>u/2,S_{N_t,-k}\le u/4\right\}.
$$

Then, by dominated convergence, we get

$$
\begin{aligned}
&\mathbb{E}\left[\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\inf_{0\le s<t}u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0,\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>u/2\right\}}\right]\\
&=\mathbb{E}\left[\sum_{k=1}^{N_t}\overline{F}\left(\frac{u+ct-\sum_{i=1}^{N_t}\mu(\mathcal{T}_i)X_i}{\mu(t)}\right)1_{\left\{\inf_{0\le s<t}u+cs-\sum_{i=1}^{N_s}\mu(\mathcal{T}_i)X_i>0,A_k\right\}}\right]+\mathcal{O}(F(u)^2)\\
&=\mathbb{E}\left[\sum_{k=1}^{N_t}\int_{\frac{u/2-S_{N_t,-k}}{\mu(\mathcal{T}_k)}}^{\inf_{\mathcal{T}_k\le s\le t}\frac{u+cs-S_{N_s,-k}}{\mu(\mathcal{T}_k)}}\overline{F}\left(\frac{u+ct-S_{N_t,-k}-\mu(\mathcal{T}_k)y}{\mu(t)}\right)f(y)\mathrm{d}y1_{\left\{S_{N_t,-k}\le u/4\right\}}\right]+\mathcal{O}(F(u)^2)\\
&=\mathbb{E}\left[\sum_{k=1}^{N_t}\frac{1}{\mu(\mathcal{T}_k)}\int_{\sup_{\mathcal{T}_k\le s\le t}c(t-s)-(S_{N_s,-k}-S_{N_s,-k})}^{u/2+ct}\overline{F}\left(\frac{y}{\mu(t)}\right)f\left(\frac{u+ct-S_{N_t,-k}+x}{\mu(\mathcal{T}_k)}\right)\mathrm{d}y1_{\left\{S_{N_t,-k}\le u/4\right\}}\right]\\
&\quad+\mathcal{O}(F(u)^2)\\
&=f(u)\mathbb{E}\left[\sum_{k=1}^{N_t}\mu(\mathcal{T}_k)^\alpha\mu(t)\int_{\sup_{\mathcal{T}_k\le s\le t}\frac{c(t-s)-(S_{N_s,-k}-S_{N_s,-k})}{\mu(t)}}^{\infty}\overline{F}\left(y\right)\mathrm{d}y\right]+o(f(u))+\mathcal{O}(F(u)^2).
\end{aligned}
$$

□

Corollary 3.2. *Let $X_1,X_2,\dots$ be iid random variables with distribution function F that is regularly varying of index $\alpha>1$ and has regularly varying density f. Further let $0<\mu_m\le\mu(t)\le\mu_M$ be a bounded function (on $[0,T]$) then*

$$
\begin{aligned}
&\psi(u,T)-\mathbb{E}\left[N_T\right]\int_0^T\mathbb{P}(X_1>u/\mu(x))\frac{\lambda(x)}{\Lambda(T)}\mathrm{d}x\\
&\lesssim f(u)\int_0^T\lambda(t)\left(\mu(t)\mathbb{E}\left[X_1\right]\int_0^t\mu(y)^\alpha\lambda(y)\mathrm{d}y+\mu(t)^\alpha\left(\mathbb{E}\left[X_1\right]\int_0^t\mu(x)\lambda(x)\mathrm{d}x-ct\right)\right)\mathrm{d}t
\end{aligned}
$$

and

$$\psi(u,T) - \mathbb{E}\left[N_T\right]\int_0^T \mathbb{P}(X_1 > u/\mu(x))\frac{\lambda(x)}{\Lambda(T)}\mathrm{d}x$$
$$\gtrsim f(u)\int_0^T \lambda(t)\mu(t)\left(\frac{ct}{\mu(t)}\overline{F}\left(\frac{ct}{\mu(t)}\right) + \mathbb{E}\left[X_1 1_{\left\{X_1 > \frac{ct}{\mu(t)}\right\}}\right]\right)\int_0^t \mu(y)^\alpha\lambda(y)\mathrm{d}y\mathrm{d}t$$
$$+ f(u)\int_0^T \lambda(t)\mu(t)^\alpha\left(\mathbb{E}\left[X_1\right]\int_0^t \mu(x)\lambda(x)\mathrm{d}x - ct\right)\mathrm{d}t.$$

Proof. Just note that

$$0 \le \sup_{\mathcal{T}_k \le s \le t}\frac{c(t-s) - (S_{N_s,-k} - S_{N_s,-k})}{\mu(t)} \le \frac{ct}{\mu(t)}$$

and

$$\int_a^\infty \overline{F}(x)\mathrm{d}x = a\overline{F}(a) + \mathbb{E}\left[X_1 1_{\{X_1 > a\}}\right].$$

□

As already mentioned, the upper bound obtained in Corollary 3.2 will be used to approximate the ruin probability in the following section where two examples will be presented in the purpose of estimating the cost of climate change for an insurance company in a fictitious portfolio.

4 Some examples

Climate change is changing the risk of the insurance portfolio and, as a consequence, insurance companies have to adjust their solvency capital over time in order to fulfill solvency requirements. In other words, climate change is representing a cost for insurance companies that we would like to evaluate on some examples thanks to the theoretical results on ruin probability obtained in Section 3.

Example 1. We consider a model where the X_i's are Pareto distributed rv's whose survival function can be written as

$$\overline{F}(x) = (1+x)^{-\alpha}.$$

We take $\alpha = 2.5$. We assume that the intensity of the non homogeneous Poisson process $\lambda(t)$ is equal to the multiplicative factor $\mu(t)$ and exhibits a cyclic behavior (corresponding to seasonal patterns) as well as a long term evolutionary trend. This trend should represent an increasing risk because of climate change. In practice the trend should be chosen on expected change in risk because of climate change. We take

$$\mu(t) = \lambda(t) = \frac{1}{4} + \frac{1}{10}\sin\left(2\pi\frac{t}{T}\right) + \frac{0.075}{120}\left(\frac{t}{T}\right)^2$$

so that the claim size distribution is changing over time, not only the frequency. More precisely, we are representing cases where the higher the frequency is, the higher the losses are. As already explained in the introduction, this kind of behavior is quite common for hurricanes for example. We will compare this model with a similar model without a trend where the intensity function includes a trigonometric component but not a polynomial one:

$$\mu^*(t) = \lambda^*(t) = \frac{1}{4} + \frac{1}{10}\sin\left(2\pi\frac{t}{T}\right).$$

This may correspond to the case where we admit there is no climate change. See Figure 2 for a plot of $\mu(t)$ (respectively $\lambda(t)$) and $\mu^*(t)$ (respectively $\lambda^*(t)$).

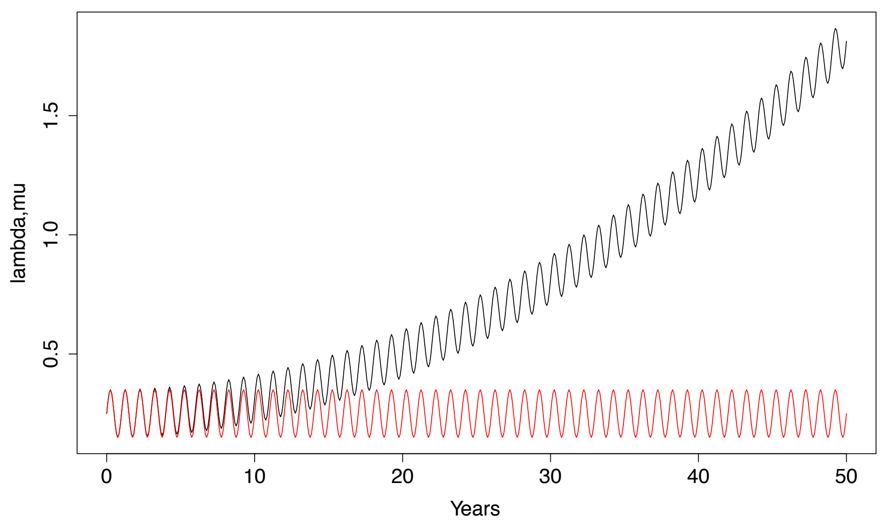

Figure 2: $\lambda(t)$ (respectively $\mu(t)$) with (in black) and without (in red) a trend.

In this paper we assume that the insurance company works on a yearly basis. Let us denote by T_0 the starting time of the year. We take $T = 12$, corresponding to the number of months in a year. Note that the expected claim in one year is given by

$$\mathbb{E}\left[\sum_{i=N_{T_0}+1}^{N_{T+T_0}} \mu(T_0 + \mathcal{T}_i)X_i\right] = \mathbb{E}\left[X_1\right]\int_{T_0}^{T+T_0} \mu(T_0+t)\lambda(T_0+t)\mathrm{d}t.$$

See Figure 3 for a plot of the expected claim amount as a function of time both with and without a trend. Obviously, because of the assumptions in our risk model, we observe an increase in the expected claim amount over the time when we consider a trend.

We assume that the insurance company is collecting risk-commensurate premiums at a constant rate c per unit time where

$$c = (1+\theta)\frac{1}{T}\mathbb{E}\left[\sum_{i=N_{T_0}+1}^{N_{T+T_0}} \mu(T_0 + \mathcal{T}_i)X_i\right]$$

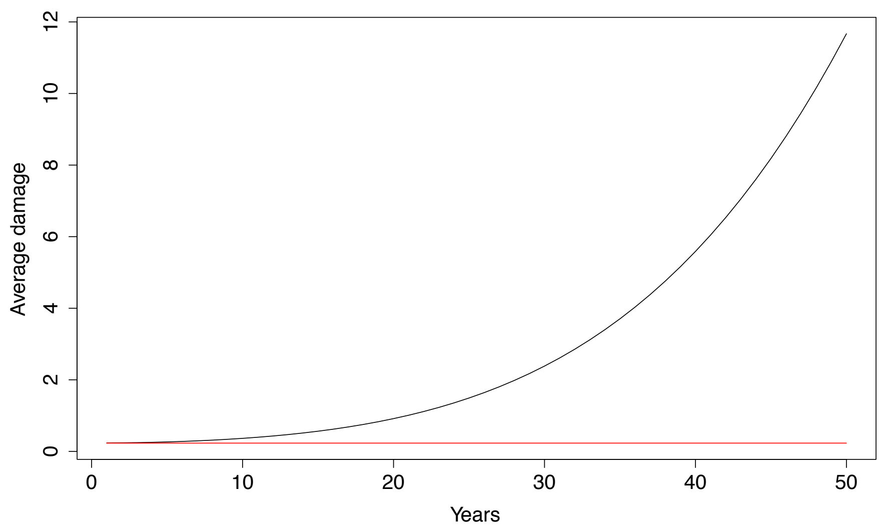

Figure 3: Average claim amount for the different years with (in black) and without (in red) a trend.

and θ is the relative safety loading. We take $\theta = 10\%$ in our example. Clearly, premiums are adjusted at the beginning of every year and, because of climate change, are increasing over time. We assume that customers are ready to accept to pay increasing premiums as premiums are commensurate to risk. But increasing claim amounts also demand to adjust risk capital every beginning of the year.

The needed solvency capital u for a year is estimated so that the finite time ruin probability is small enough, i.e. $\psi(u, T_0, T_0 + T) = 1/200$. As an approximation for the ruin probability we will use the asymptotic upper bound given in Corollary 3.2. To evaluate the needed capital for later years we just start the process at the starting time of the considered year and recalculate the premium. This provides us with a series of needed solvency capital for the future. The resulting solvency capitals for the different years are represented in Figure 4.
Moreover, in Figure 5 we plot the relative solvency capital, i.e the needed solvency capital over the expected claim amount. We can observe that, in case of trend, the relative solvency capital is decreasing over time. A possible explanation is that with more claims there is a bigger diversification effect that reduces the relative solvency capital.

To sum up, an insurance company has to provide solvency capital every year and, in a model with trend, the solvency capital is increasing every year. Clearly, insurance companies have to follow. The increase of the solvency capital can only partly be financed by the earnings through safety loadings, hence extra money has to be inserted. This needed extra money can be seen as the true cost of the trend for an insurance company and will be denoted as cost of trend in the rest of the

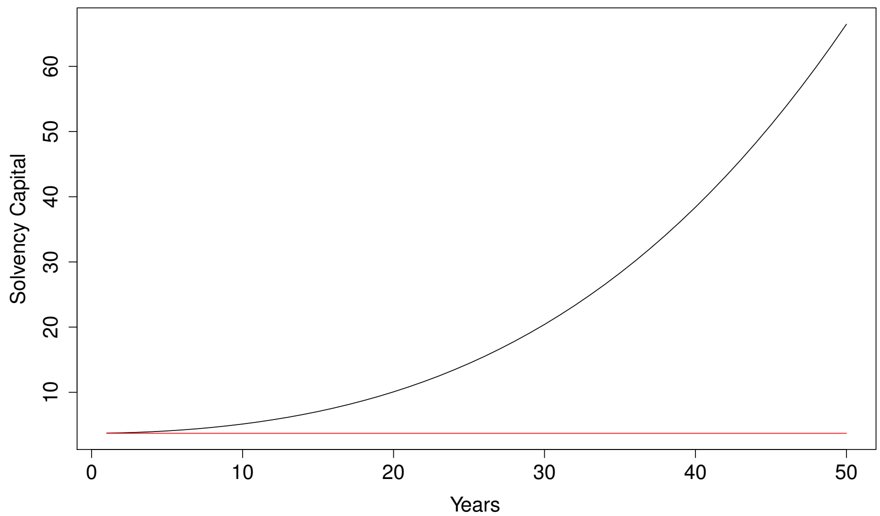

Figure 4: Solvency capital needed at the beginning of every year with (in black) and without (in red) a trend.

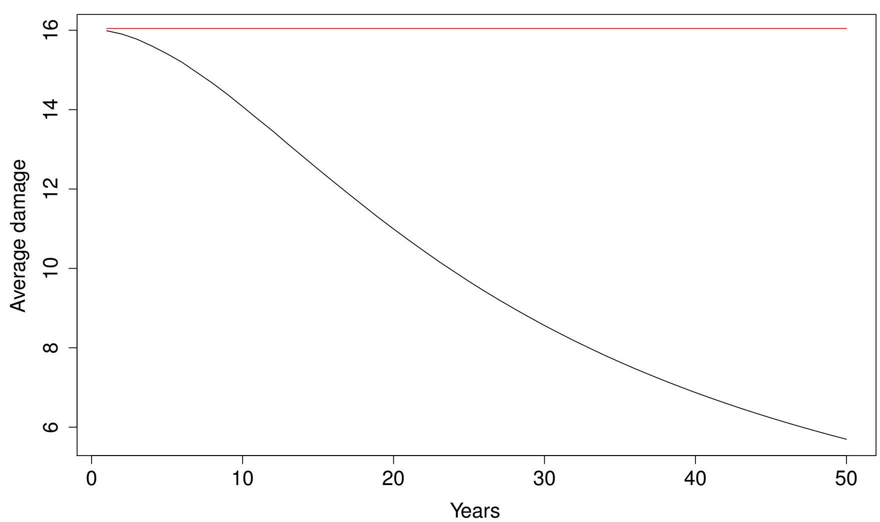

Figure 5: Solvency capital needed at the beginning of every year over the expected claim amount, with (in black) and without (in red) a trend.

paper. The accumulated cost of trend is the difference between the solvency capital with trend, the solvency capital without trend and the collected premiums times the safety loading. The cost of climate change for a year can then be evaluated as the accumulated cost of climate change for the year minus the accumulated cost of climate change of the previous year.

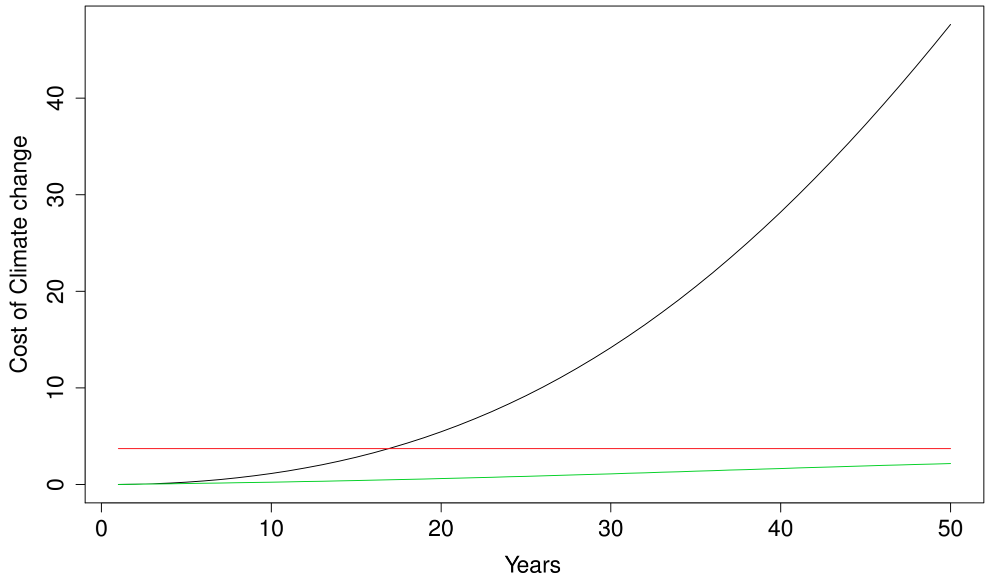

Figure 6: Accumulated cost of trend (black line), solvency capital without trend (red line), and the cost of trend (green line).

The accumulated cost of trend is represented in Figure 6 (black line) together with the solvency capital without trend (red line) and the cost of trend (green line). We can observe that the cost of trend is increasing over time but stays smaller than the solvency capital without a trend. Finally we want to relate the yearly cost of trend to the expected claim amount. In Figure 7 we provide the yearly cost of trend over the expected claim amount together with the collected premiums over the expected claim amount. We can observe that the relative cost increases in the beginning but decreases after approximately 15 years. The cost of trend can reach up to 40% of the expected claim amount which is much higher than the chosen safety loading (red line). So we can conclude that, in this specific example, the cost of trend is a significant cost for insurance companies.

Example 2 (about flood risk). In this example we will focus on flood hazard in France. Among flood hazards, river floods are considered one of the most important natural disasters in Europe whose impact can be devastating. We will use results by Alfieri et al. (2015) [1] to obtain projections of changes in the frequency of flood hazard in France through the current century. Since we did not find usable informations on projected changes in claim size related to flood hazard in France, we decided to assume that the distribution of claim sizes will stay constant over time. On the other hand, the claim frequency will be supposed to increase in a way that we are going to precise. Alfieri et al. (2015) [1], investigate changes in the frequency of extreme events with return periods equal or larger than 100 years, in three time

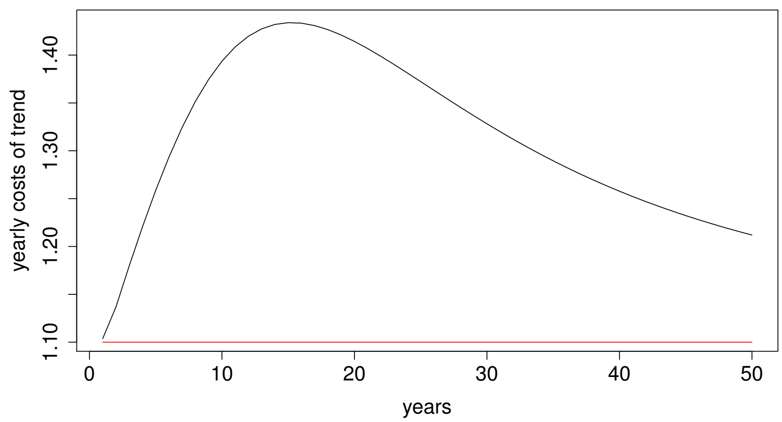

Figure 7: Yearly cost of trend over the expected claim amount (black line) and collected premiums over expected claim amount (red line).

slices: 2006 - 2035, 2036 - 2065 and 2066 - 2095 (referred to as 2020, 2050 and 2080 respectively). They state that, in France, the frequency of a flood with return period 100 years can be illustrated as in the table below (cf. Table 2 in Alfieri et al. (2015) [1]):

1990	2020	2050	2080
0.0094	0.0213	0.0238	0.0324

Table 1: Mean annual exceedance frequency of the 100-year return period peak flow for France.

We will use a linear approximation of the frequencies in Table 1 in order to obtain an intensity function, $\lambda(t) = -0.463 + 0.000238 * (1990 + t)$, corresponding to the projected increase in frequencies. Again, we assume that the X_i's are Pareto distributed rv's with $\alpha = 2.5$. Since we are only considering events with a return period of at least 100 years we will use the conditional distribution given that we are bigger than the 100 year event i.e we will use $x > x_0$ with $x_0 = 0.01^{-1/2.5} - 1 \approx 5.3$. The survival function of X can then be written as:

$$\overline{F}(x) = 100(1 + x)^{-2.5}, \qquad x > x_0.$$

Moreover we will assume that in the model without trend, both the intensity function $\lambda(t)$ and the multiplicative function $\mu(t)$ are constant. More precisely, $\lambda^*(t) = 0.01$ and $\mu^*(t) = 1$. Remark that we also assume $\mu(t) = \mu^*(t)$ so that claim size is not time dependent in this example. We will use $T = 1$. Other assumptions required for the calculation of quantities of interest, are the same as in Example 1.

Because of the assumptions in our example, the solvency capital needed at the beginning of every year is increasing over time in case of trend (see Figure 8).

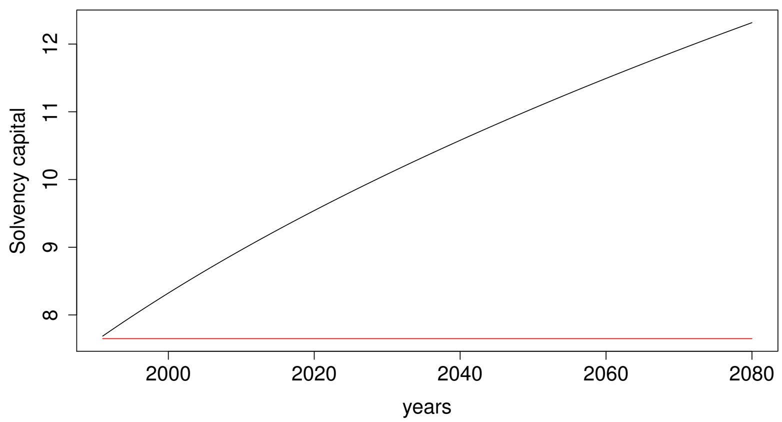

Figure 8: Solvency capital needed at the beginning of every year with (in black) and without (in red) a trend.

On the other hand, the relative solvency capital (solvency capital needed at the beginning of every year over the expected claim amount) in case of trend is decreasing because of possible diversification effects (see Figure 9). Again, the accumulated cost of trend is increasing but it is, in this example, smaller than the needed solvency capital (see Figure 10). Further, the cost of trend is slightly decreasing. Finally, in Figure 11, we can observe that the relative cost is increasing at the beginning but then immediately decreasing. Anyhow, it is higher than the safety loading, so representing a significant cost for the insurance company.

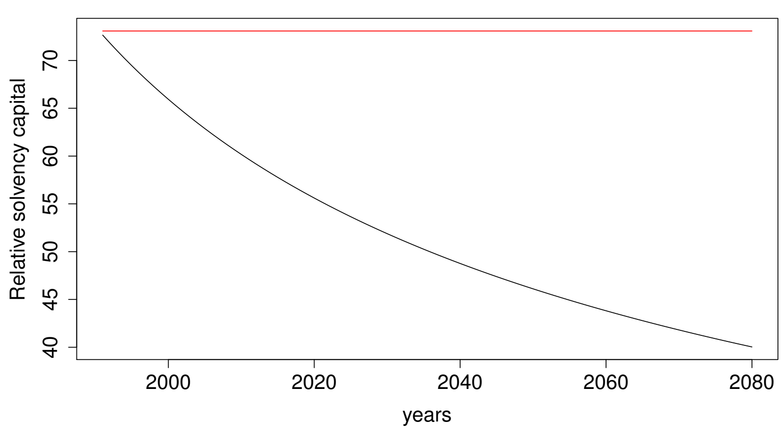

Figure 9: Solvency capital needed at the beginning of every year over the expected claim amount, with (in black) and without (in red) a trend.

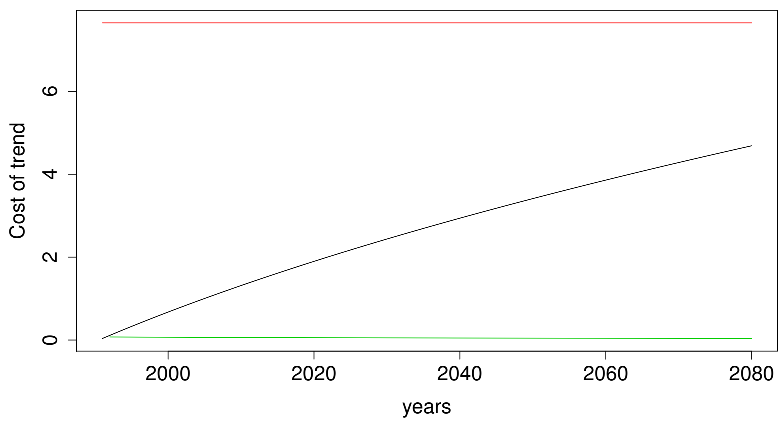

Figure 10: Accumulated cost of trend (black line), solvency capital without trend (red line), and the cost of trend (green line).

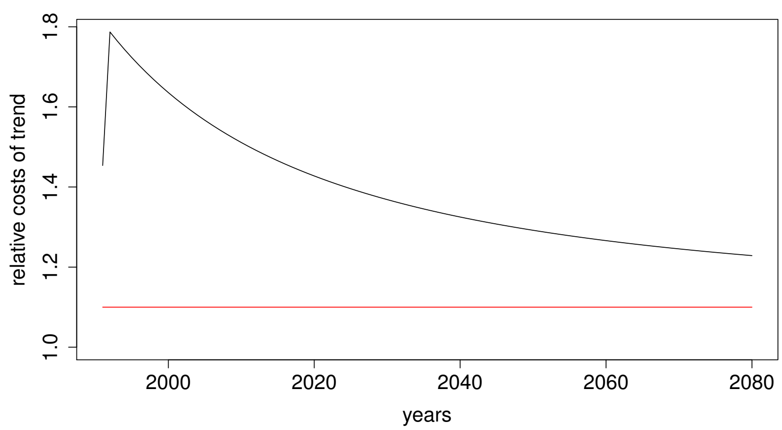

Figure 11: Yearly cost of trend over the expected claim amount (black line) and collected premiums over expected claim amount (red line).

5 Conclusions

In this paper a generalization of the classical risk model has been proposed to better represent more realistic cases where weather-related claims not only present a seasonal pattern but are also affected by climate change. The model we propose is essentially a non-homogeneous Poisson process with severity depending on time. This allows for environments where the claim size is higher or lower depending on the season. For example, we noted that the most expensive hurricanes occur in the peak season. Our model allows to take into account this kind of trend in weather events as well as other kinds of events. Under the proposed risk model, we obtain asymptotics for finite time ruin probability. Theoretical results are then used to illustrate on some examples how much climate change would cost to an insurance company who will experience events, like tropical storms or floods, which are more and more frequent and whose consequences are heavier and heavier. In our simplified portfolios, the cost of climate change seems to be significant for insurance companies. This field seems worth exploring on more complex models.

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